1) a) $\int_1^2 \frac{1+6x^2}{x+2x^3} \cdot dx$

$\boxed{z = x+2x^3}$ Substitution

$\frac{dz}{dx} = 1+6x^2$ $\boxed{dx = \frac{1}{1+6x^2} \cdot dz}$

$\int_1^2 \frac{\cancel{1+6x^2}}{z} \cdot \frac{1}{\cancel{1+6x^2}} \cdot dz$

$$\int_1^2 \frac{1}{z} \cdot dz \qquad \boxed{\int \frac{1}{x}\, dx = \ln(x)}$$

$$\Big[\ln(z)\Big]_1^2$$

Rücksubstitution

$$\boxed{z = x + 2x^3}$$

$$\Big[\ln(x+2x^3)\Big]_1^2 = \ln(2+2\cdot 2^3) - \ln(1+1\cdot 1^3) =$$

$$\ln(18) - \ln(3) = \ln\left(\frac{18}{3}\right) = \underline{\underline{\ln(6)}}$$

b) $\int_{0}^{\pi} \sin^2(x) \cdot dx = \int_{0}^{\pi} \sin(x) \cdot \sin(x) \cdot dx$

Partielle Integration:

$$\int_{0}^{\pi} u(x) \cdot v'(x) \cdot dx = \left[u(x) \cdot v(x)\right]_0^{\pi} - \int_{0}^{\pi} u'(x) \cdot v(x) \cdot dx$$

$u(x) = \sin(x) \qquad v'(x) = \sin(x)$

$u'(x) = \cos(x) \qquad v(x) = -\cos(x)$

$$\int_{0}^{\pi} \sin(x) \cdot \sin(x) \cdot dx = \left[\sin(x) \cdot -\cos(x)\right]_0^{\pi} - \int_{0}^{\pi} \cos(x) \cdot -\cos(x) \cdot dx$$

$$\int_0^{\pi} \sin(x)\,\sin(x)\cdot dx = \Big[-\sin(x)\cos(x)\Big]_0^{\pi} + \int_0^{\pi} \cos^2(x)\cdot dx$$

Es gilt: $\sin^2(x) + \cos^2(x) = 1$

$\longrightarrow \cos^2(x) = 1 - \sin^2(x)$

$$\int_0^{\pi} \sin^2(x)\cdot dx = \Big[-\sin(x)\cdot\cos(x)\Big]_0^{\pi} + \int_0^{\pi} \big(1 - \sin^2(x)\big)\cdot dx$$

$$\int_0^{\pi} \sin^2(x)\cdot dx = \Big[-\sin(x)\cdot\cos(x)\Big]_0^{\pi} + \int_0^{\pi} 1\cdot dx - \int \sin^2(x)\,dx$$

$$2 \cdot \int_0^\pi \sin^2(x) \cdot dx = \left[-\sin(x)\cdot\cos(x)\right]_0^\pi + \left[x\right]_0^\pi \quad \Big| :2$$

$$\int_0^\pi \sin^2(x) \cdot dx = \left[\frac{-\sin(x)\cdot\cos(x)+x}{2}\right]_0^\pi$$

$$= \frac{-\sin(\pi)\cdot\cos(\pi)+\pi}{2} - 0 = 1{,}573$$

2 a)

$$\int \frac{\ln(x)}{x^2} \cdot dx$$

$$\int u(x) \cdot v'(x) \cdot dx = u(x)\, v(x) - \int u'(x) \cdot v(x) \cdot dx$$

$u(x) = \ln(x)$ $\qquad v'(x) = \frac{1}{x^2} \rightarrow \int \frac{1}{x^2} = \int x^{-2} = \frac{1}{-2+1} x^{-2+1} = -x^{-1} = -\frac{1}{x}$

$u'(x) = \frac{1}{x}$ $\qquad v(x) = -\frac{1}{x}$

$$\int \ln(x) \cdot \frac{1}{x^2} \cdot dx = \ln(x) \cdot -\frac{1}{x} - \int \frac{1}{x} \cdot -\frac{1}{x} \cdot dx$$

$$\int \ln(x) \cdot \frac{1}{x^2} \cdot dx = -\frac{\ln(x)}{x} + \int \frac{1}{x^2} \cdot dx$$

$$\int \ln(x) \frac{1}{x^2} dx = -\frac{\ln(x)}{x} + \left(-\frac{1}{x}\right) + C$$

$$\int \ln(x) \cdot \frac{1}{x^2} \cdot dx = -\frac{1}{x} \left(\ln(x) + 1 \right) + C$$

2 b) $\int \cos(\ln(x)) \cdot dx$

Substitution:

$$\boxed{z = \ln(x)}$$

$$\frac{dz}{dx} = \frac{1}{x} \quad \Leftrightarrow \quad \boxed{dx = x \cdot dz}$$

$$\int \cos(z) \cdot \boxed{x} \; dz$$

$$z = \ln(x) \longleftrightarrow \boxed{x = e^z}$$

$$\int \cos(z)\, e^z \cdot dz$$

Partielle Integration:

$$\int u(z) \cdot v'(z) \cdot dz = u(z)\, v(z) - \int u'(z) \cdot v(z) \cdot dz$$

$$u(z) = \cos(z) \qquad v'(z) = e^z$$

$$u'(z) = -\sin(z) \qquad v(z) = e^z$$

$$\int \cos(z) \cdot e^z \cdot dz = \cos(z) \cdot e^z - \int -\sin(z) \cdot e^z \cdot dz$$

$$\int \cos(z) \cdot e^z \cdot dz = \cos(z) \cdot e^z + \int \sin(z) \cdot e^z \cdot dz$$

$u(z) = \sin(z) \qquad v'(z) = e^z$

$u'(z) = \cos(z) \qquad v(z) = e^z$

} 2. partielle Integration

$$\int \cos(z) \cdot e^z \cdot dz = \cos(z) \cdot e^z + \sin(z) \cdot e^z - \int \cos(z) \cdot e^z \cdot dz \quad \Big| + \int \cos(z) \cdot e^z \, dz$$

$$2 \int \cos(z)\, e^z \cdot dz = \cos(z) \cdot e^z + \sin(z) \cdot e^z \quad \Big| : 2$$

$$\int \cos(z) \cdot e^z \cdot dz = \frac{1}{2} \cos(z) \cdot e^z + \frac{1}{2} \cdot \sin(z)\, e^z + C$$

<u>Rücksubstitution</u>:

$z = \ln(x) \qquad x = e^z$

$$\int \cos(\ln(x)) \cdot dx = \frac{1}{2} \cdot \cos(\ln(x)) \cdot x + \frac{1}{2} \cdot \sin(\ln(x)) \cdot x + C$$