1) a)

$$A+B=\begin{pmatrix} 4+5 & 2+-2 & 5+3 \\ 11+-3 & 3+4 & -2+1 \\ 0+5 & 2+1 & 3+-1 \end{pmatrix}=\begin{pmatrix} 9 & 0 & 8 \\ 8 & 7 & -1 \\ 5 & 3 & 2 \end{pmatrix}$$

$$A-B=\begin{pmatrix} -1 & 4 & 2 \\ 14 & -1 & -3 \\ -5 & 1 & 4 \end{pmatrix}$$

$A \cdot B =$

$$A = \begin{pmatrix} \boxed{4 \quad 2 \quad 5} \\ 1 & 3 & -2 \\ 0 & 2 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & -2 & 3 \\ -3 & 4 & 1 \\ 5 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 39 & 5 & 9 \\ -14 & 8 & 8 \\ 9 & 11 & -1 \end{pmatrix}$$

$4 \cdot 5 + 2 \cdot (-3) + 5 \cdot 5 = 39$

$4 \cdot -2 + 2 \cdot 4 + 5 \cdot 1 = 5$

$4 \cdot 3 + 2 \cdot 1 + 5 \cdot -1 = 9$

$$B \cdot A = \begin{pmatrix} 18 & 10 & 38 \\ -8 & 8 & -20 \\ 21 & 11 & 20 \end{pmatrix}$$

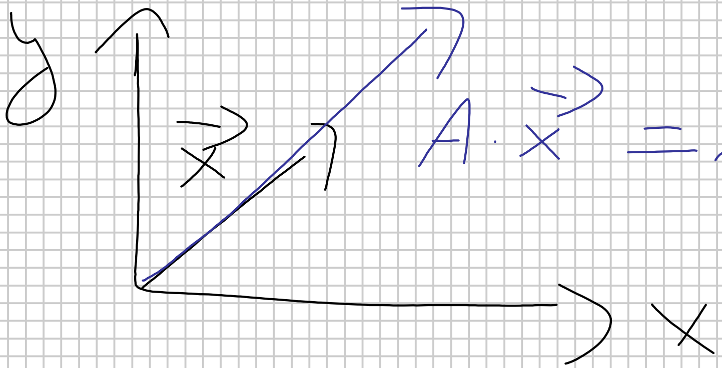
b) $A = \begin{pmatrix} 5 & -3 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$

$$A \cdot B = -13$$

$B \cdot A =$

$B = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} \qquad A = \begin{pmatrix} 5 & -3 & 2 \end{pmatrix}$

$$B \cdot A = \begin{pmatrix} 10 & -6 & 4 \\ 25 & -15 & 10 \\ -20 & 12 & -8 \end{pmatrix}$$




$$A \cdot \vec{x} = \lambda \cdot \vec{x}$$

$\lambda$ = Eigenwert

$\vec{x}$ = Eigenvektor

$$\boxed{\det(A - \lambda E) = 0}$$

$$A = \begin{pmatrix} 1 & 5 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 2 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda E = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$A - \lambda E = \begin{pmatrix} 1-\lambda & 5 & 4 \\ 0 & 2-\lambda & 0 \\ 0 & 3 & 2-\lambda \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 1-\lambda & 5 & 4 \\ 0 & 2-\lambda & 0 \\ 0 & 3 & 2-\lambda \end{vmatrix}$$

$$+ \qquad \begin{pmatrix} 1-\lambda & 5 & 4 \\ 0 & 2-\lambda & 0 \\ 0 & 3 & 2-\lambda \\ 1-\lambda & 5 & 4 \\ 0 & 2-\lambda & 0 \end{pmatrix}$$

$$-$$

$$(1-\lambda)\cdot(2-\lambda)\cdot(2-\lambda) + \cancel{0\cdot 3\cdot 4} + \cancel{0\cdot 5\cdot 0}$$

$$-\cancel{4\cdot(2-\lambda)\cdot 0} - \cancel{0\cdot 3\cdot(1-\lambda)} - \cancel{(2-\lambda)\cdot 5\cdot 0}$$

$$\det(A-\lambda E) = 0$$

$$(1-\lambda)\cdot(2-\lambda)\cdot(2-\lambda) = 0$$

$$\boxed{\lambda = 1 \qquad \lambda = 2}$$

## Eigenvektor

$$(A - \lambda E) \cdot \vec{x} = \vec{0}$$

$$\boxed{\lambda = 1}$$

$$(A - 1 \cdot E) \cdot \vec{x} = \vec{0}$$

$$A - E = \begin{pmatrix} 1-1 & 5 & 4 \\ 0 & 2-1 & 0 \\ 0 & 3 & 2-1 \end{pmatrix} = \begin{pmatrix} 0 & 5 & 4 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$$

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{pmatrix} 0 & 5 & 4 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

Gauß-Algorithmus

$(1) - (3) \cdot 4$

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{pmatrix} 0 & -7 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \qquad (1) + (2) \cdot 7$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \qquad (3) - (2) \cdot 3$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$x_2 = 0 \rightarrow x_1 = 1 \quad$ da, $\vec{x} \neq 0$

$x_3 = 0$

$$E_i g(\lambda = 1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\boxed{(A - \lambda E) \cdot \vec{x} = 0}$$

$\lambda = 2$

$$(A - 2 \cdot E) \cdot \vec{x} = 0$$

$$2 \cdot E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$(A - 2 \cdot E) = \begin{pmatrix} -1 & 5 & 4 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$$

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{pmatrix} -1 & 5 & 4 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$(3) : 3$

Gauß-Algorithmus

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \begin{pmatrix} -1 & 5 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad (1) - (3) \cdot 5$$

$$\begin{pmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-x_1 + 4 \cdot x_3 = 0 \rightarrow x_3 = \frac{1}{4} x_1$$

$$x_2 = 0$$

$$\boxed{x_3 = 1 \qquad 1 = \frac{1}{4} x_1 \qquad \underline{\underline{x_1 = 4}}}$$

$$x_3 = 3 \qquad 3 = \frac{1}{4} x_1 \qquad x_1 = 12$$

$$E_g(\lambda = 2) = \boxed{\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}} = \boxed{\begin{pmatrix} 12 \\ 0 \\ 3 \end{pmatrix}}$$

$$\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot 3 = \begin{pmatrix} 12 \\ 0 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} \boxed{-2 \quad -1 \quad 4 \quad 2} \\ 1 \quad 0 \quad 1 \quad 1 \\ 3 \quad 0 \quad -2 \quad -1 \\ 0 \quad 2 \quad 6 \quad 3 \end{pmatrix}$$

$j = 2$

$$\det(A) = \sum_{i,j=1}^{n} (-1)^{(i+j)} \cdot a_{ij} \cdot \det(A_{ij})$$

$$\det(A_{11}) = \det\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & -1 \\ 0 & 6 & 3 \end{pmatrix}$$

$a_{12} = -1$

$\boxed{j = 2,\ i = 1}$

$$+ \qquad \det\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & -1 \\ 0 & 6 & 3 \\ 1 & 1 & 1 \\ 3 & -2 & -1 \end{pmatrix} = 9 \qquad -$$

$$\det(A) = (-1)^{1+2} \cdot a_{12} \cdot \det(A_{12})$$

$$\boxed{\det(A) = (-1) \cdot (-1) \cdot 9}$$

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$$A_{22} = \begin{pmatrix} -2 & 4 & 2 \\ 3 & -2 & -1 \\ 0 & 6 & 3 \end{pmatrix}$$

$$a_{22} = 0$$

$$A_{32}$$

$$a_{32} = 0$$

$$\det(A_{42}) = \det\begin{pmatrix} -2 & 4 & 2 \\ 1 & 1 & 1 \\ 3 & -2 & 1 \end{pmatrix} = 4 \qquad \boxed{a_{42} = 2}$$

$$\begin{array}{ccc} -2 & 4 & 2 \\ 1 & 1 & 1 \end{array}$$

$$+ \qquad -$$

$$\det(A) = (-1)\cdot(-1)\cdot 9 + (-1)^{4+2}\cdot 2\cdot 4$$

$$= (-1)\cdot(-1)\cdot 9 + 1\cdot 2\cdot 4$$

$$= 9 + 8 = \underline{\underline{17}}$$