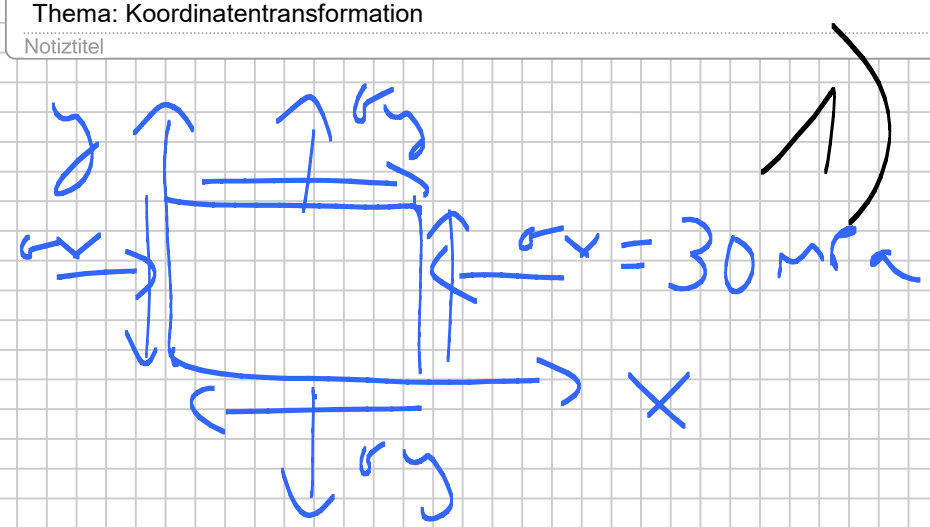
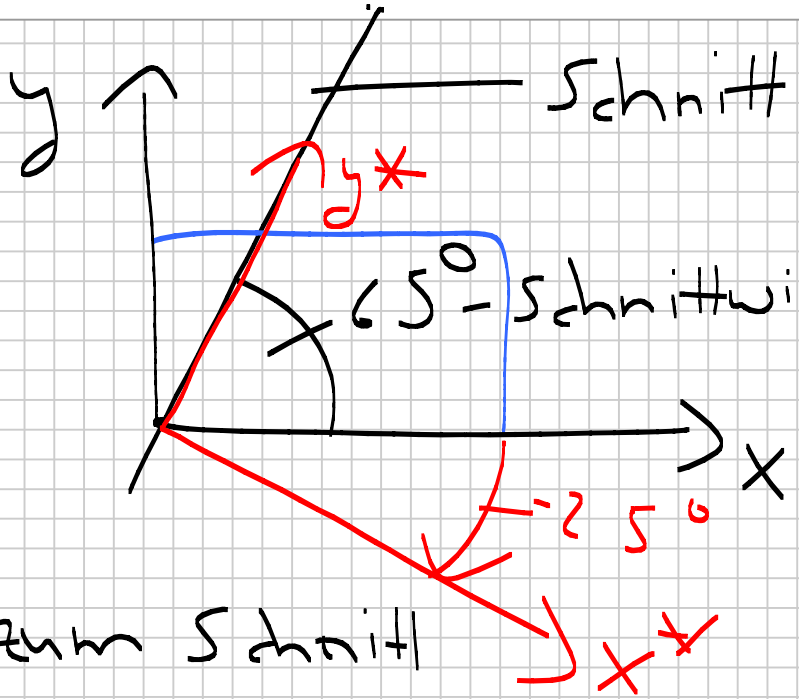
Webinar: Elastostatik
Thema: Koordinatentransformation

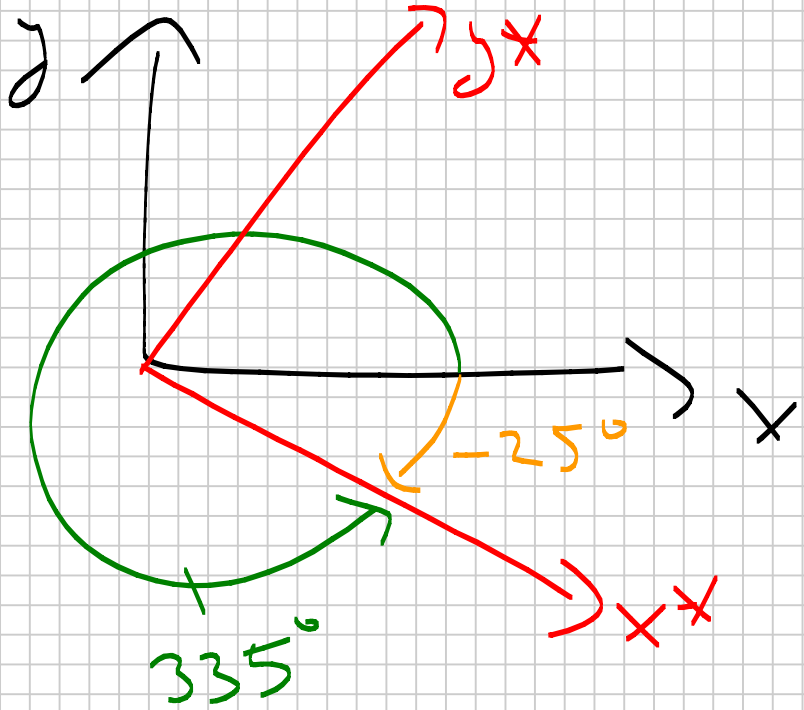
Notiztitel 05.07.2016





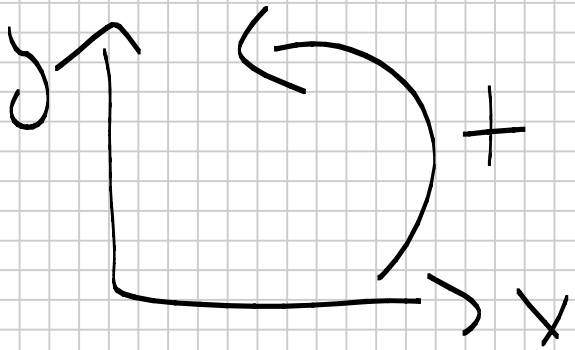
Rechtsdrehung → negativer Winkel

- y^*-Achse parallel zum Schnitt
- x^*-Achse senkrecht zum Schnitt



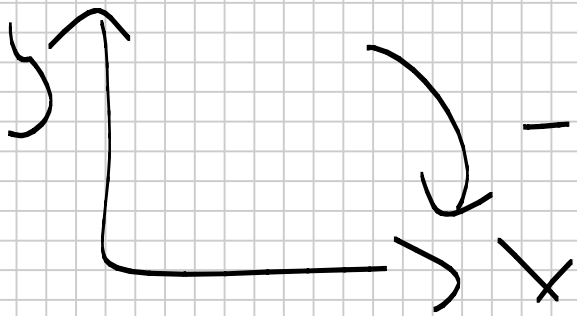
Alternative:

Statt negativem Winkel (-25°)
den positiven Winkel einsetzen
(+335°)



positive Drehrichtung

→ positiver Winkel



negative Drehrichtung

→ negativer Winkel

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos(2\alpha) + \tau_{xy} \sin(2\alpha)$$

$$\sigma_{x'} = \frac{-30 + 15}{2} + \frac{-30 - 15}{2} \cdot \cos(2 \cdot -25^\circ) + 5 \cdot \sin(2 \cdot -25^\circ)$$

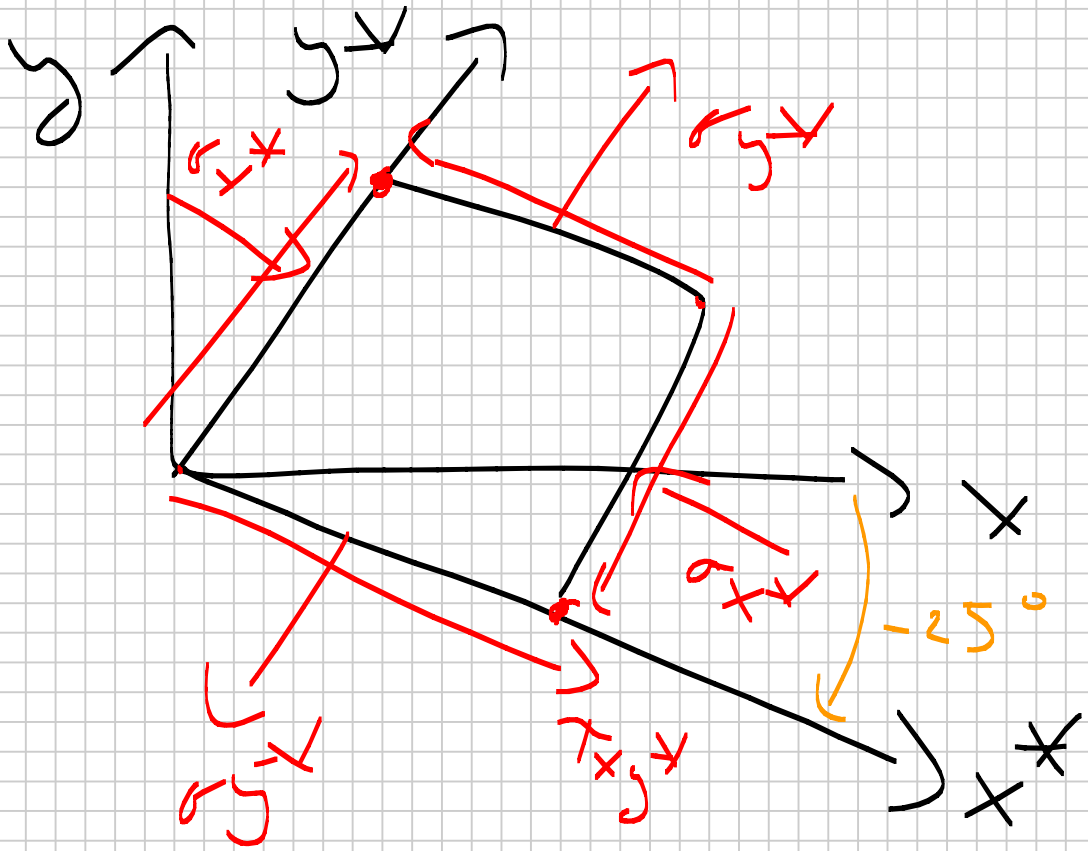
$$\boxed{\sigma_{x'} = -25{,}79\ \text{MPa}}$$

$$\sigma_y^{\prime} = \frac{-30+15}{2} + \frac{30+15}{2}\cdot\cos(2\cdot -75^\circ) - 5\cdot\sin(2\cdot -75^\circ)$$

$$\boxed{\sigma_y^{\prime} = 10{,}75\ \text{MPa}}$$

$$\tau_{xy}^{\prime} = \tau_{yx}^{\prime} = \frac{30+15}{2}\cdot\sin(2\cdot -75^\circ) + 5\cdot\cos(2\cdot -75^\circ)$$

$$\boxed{\tau_{xy}^{\prime} = \tau_{yx}^{\prime} = -14{,}02\ \text{MPa}}$$



2) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-30 + 15}{2} \pm \sqrt{\left(\frac{-30 - 15}{2}\right)^2 + 5^2}$$

$$\sigma_1 = 15{,}55\ \text{MPa}$$

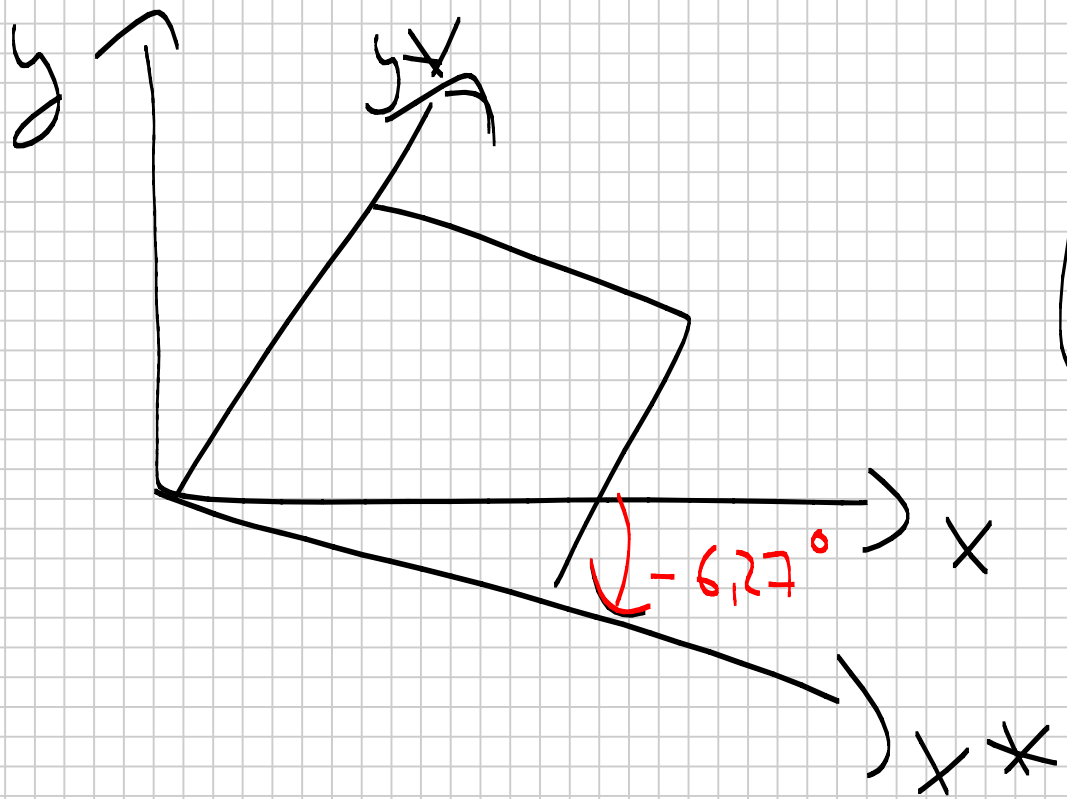
$$\sigma_2 = -30{,}55\ \text{MPa}$$

$$\tan(2\alpha^*) = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$2\alpha^* = \tan^{-1}\left(\frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$2\alpha^* = \tan^{-1}\left(\frac{2 \cdot 5}{-30 - 15}\right) = -12{,}53$$

$$\boxed{\alpha^* = -6{,}27°}$$

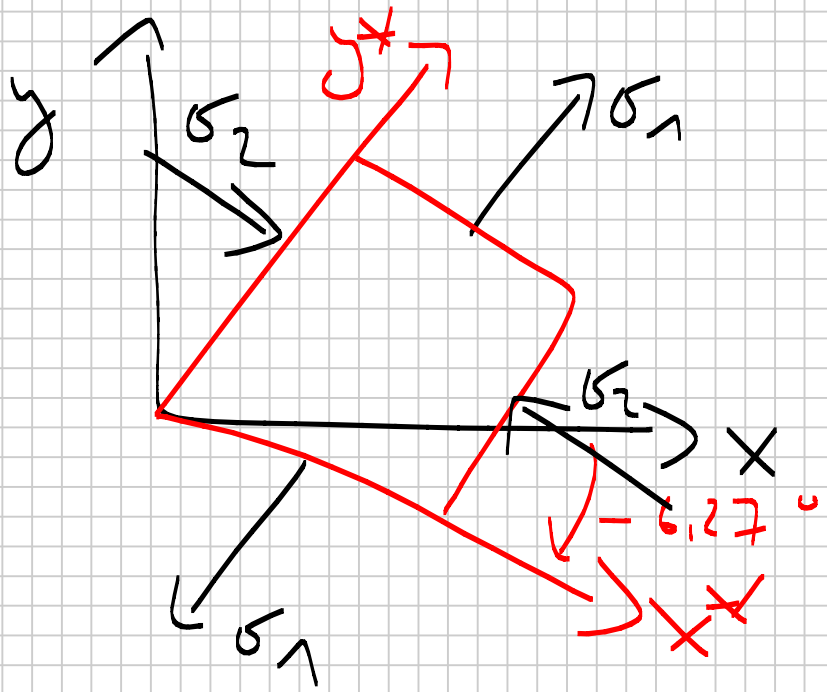


$$\tau_{xy} = \tau_{yx} = 0$$

$$\sigma_x^* = \frac{-30+15}{2} + \frac{-30-15}{2} \cdot \cos(2 \cdot -6{,}27°) + 5 \cdot \sin(2 \cdot -6{,}27°)$$

$$\boxed{\sigma_x^* = -30{,}55\ \text{MPa} = \sigma_2}$$

→ Winkel $\varphi^* = -6{,}27°$ gehört zu σ_2



$$\tau_{xy} = \tau_{yx} = 0$$

3) $$\tau_{1,2} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{1,2} = \pm \sqrt{\left(\frac{-30 - 15}{2}\right)^2 + 5^2}$$

$$\tau_1 = 23{,}05\ \text{MPa}$$

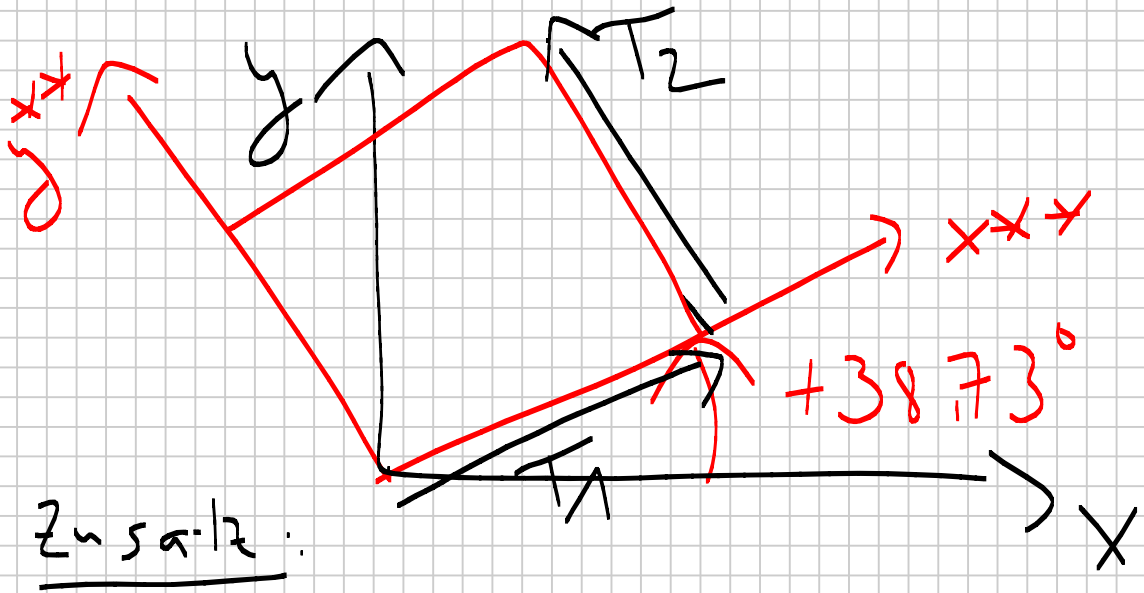
$$\tau_2 = -23{,}05\ \text{MPa}$$

$$\tan(2\alpha^{**}) = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$2\alpha^{**} = \tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

$$2\alpha^{**} = \tan^{-1}\left(-\frac{30 - 15}{2 \cdot 5}\right) = -77{,}47°$$

$$\boxed{\alpha^{**} = 38{,}74°}$$



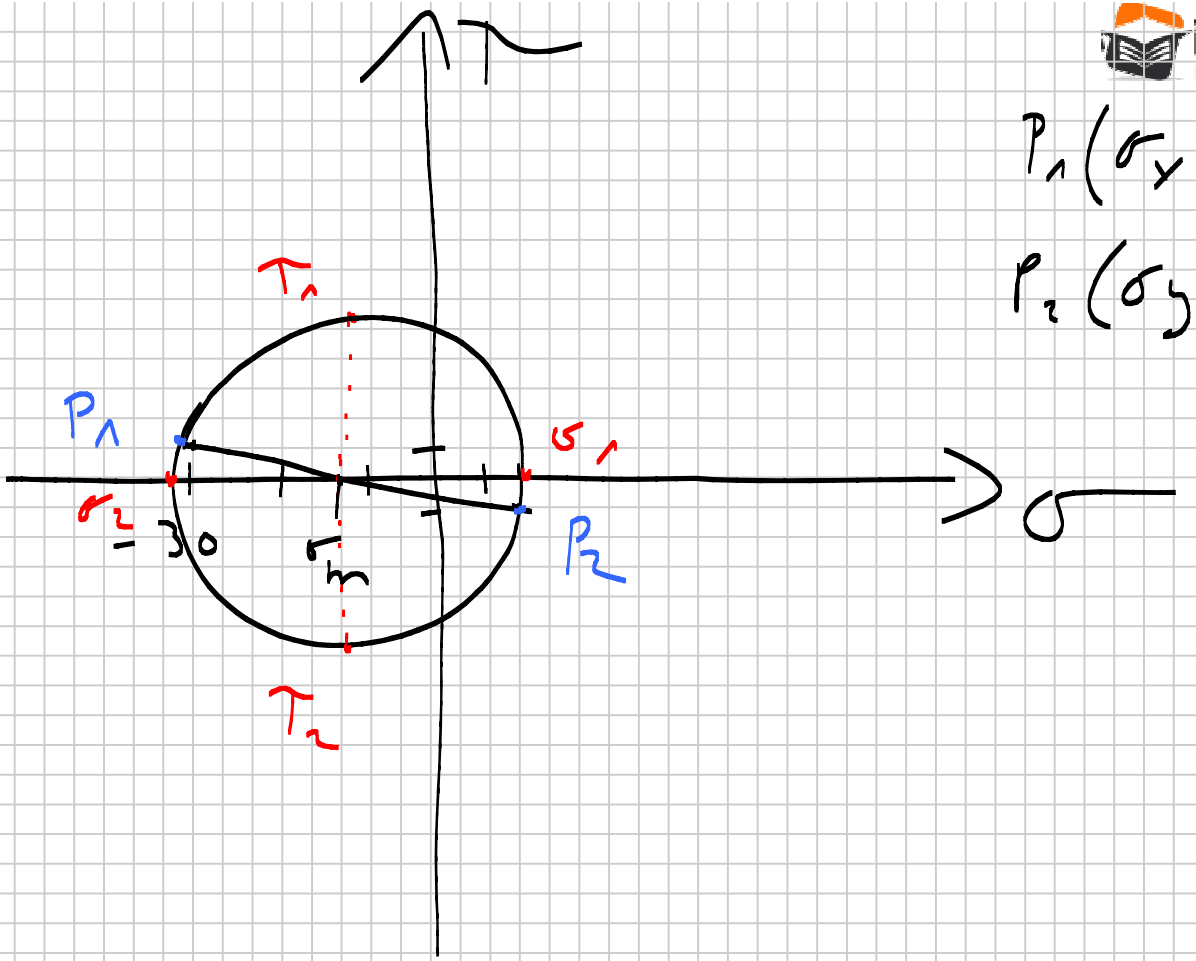
Einsetzen:

$$\tau_{x'y'} = \frac{30+15}{2} \cdot \sin(2 \cdot 38{,}73°) + 5 \cdot \cos(2 \cdot 38{,}73°)$$

$$\boxed{\tau_{x'y'} = 23{,}04\ \text{MPa} = \tau_1}$$

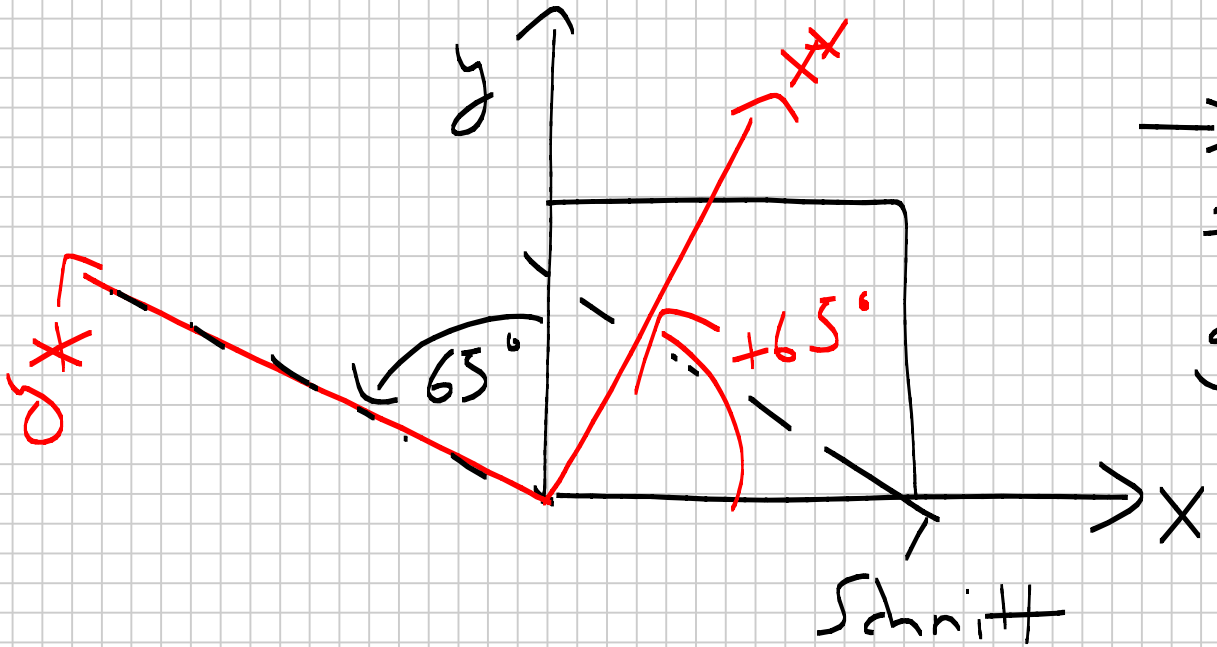
$$\sigma_m = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 15}{2} = -7{,}5\,\text{MPa}$$

→ Normalspannung nimmt mittleren Wert an bei Vorliegen der Hauptschubspannungen



$P_1(\sigma_x / \tau_{xy})$

$P_2(\sigma_y / -\tau_{xy})$



→ Schnittwinkel zur y-Achse gleichteilig Drehwinkel