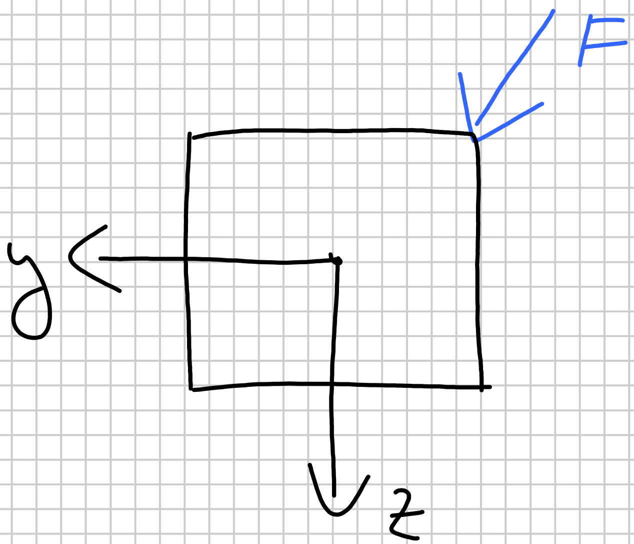
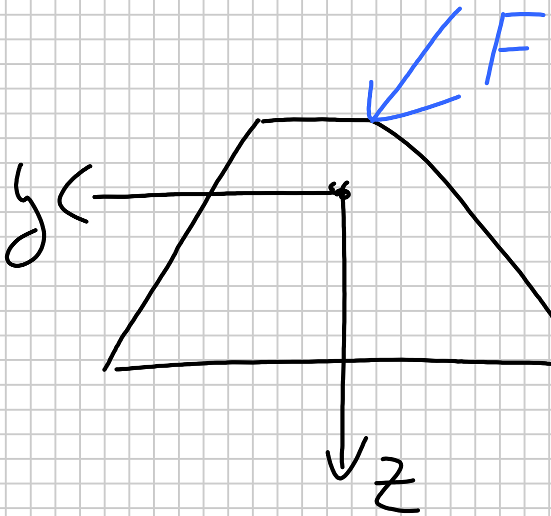
Schiefe Biegung

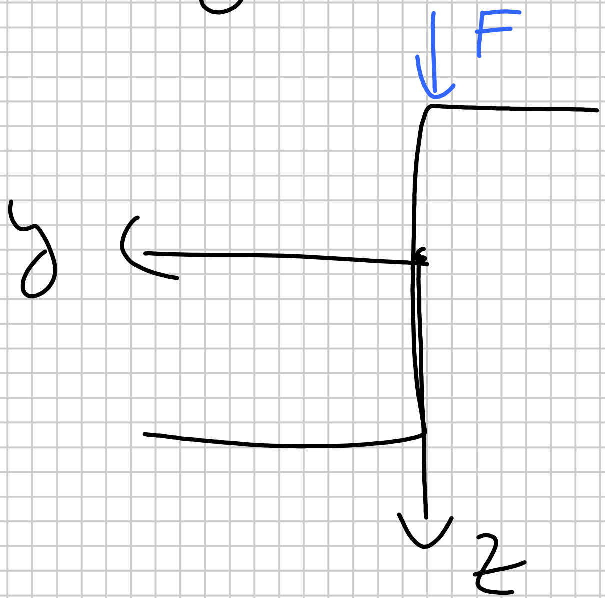
Symmetrische Querschnitte





z-y-Achsen sind Haupt-achsen

Unsymmetrische Querschnitte



z-y-Achsen sind **keine** Hauptachsen

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

$$I_y, I_z, I_{yz}$$

$$I_{y_1} = \frac{t^3 \cdot \left(a - \frac{t}{2}\right)}{12} \qquad I_{y_2} = \frac{\left(2a + t/2 + t/2\right)^3 \cdot t}{12}$$

$$I_{y_3} = \frac{t^3 \cdot \left(a - t/2\right)}{12}$$

$$I_{z_1} = \frac{(a - t/2)^3 \cdot t}{12} \qquad I_{z_2} = \frac{t^3 \cdot (2a + t/2 + t/2)}{12}$$

$$I_{z_3} = \frac{(a - t/2)^3 \cdot t}{12}$$

$$I_{yz_1} = I_{yz_2} = I_{yz_3} = 0$$

$\boxed{t << a}$ $\quad t^n \quad n > 1 \rightarrow$ eliminieren

$$I_{y3} = \frac{t^3 \cdot \left(a - t/2\right)}{12}$$

$$\frac{\cancel{t^3 \cdot a} - \cancel{t^4/2}}{12}$$

$$I_{y3} = 0$$

$I_{y_1} = 0 \qquad I_{y_2} = \frac{8a^3 \cdot t}{12} \qquad I_{y_3} = 0$

$I_{z_1} = \frac{a^3 \cdot t}{12} \qquad I_{z_2} = 0 \qquad I_{z_3} = \frac{a^3 \cdot t}{12}$

$$I_y = \sum \left(I_{y_i} + z_i^2 \cdot A_i \right)$$

$$I_z = \sum \left(I_{z_i} + y_i^2 \cdot A_i \right)$$

$z_1 = a \quad , z_2 = 0 \quad , z_3 = a$

$$y_1 = \frac{a - t/2}{2} + \frac{t}{2} = \frac{a}{2} + \frac{t}{4}$$

$$y_2 = 0$$

$$y_3 = \frac{a}{2} + \frac{t}{4}$$

$$I_y = \left(0 + a^2 \cdot (t \cdot a)\right) + \left(\frac{8a^3 \cdot t}{12} \cdot 0 \cdot (t \cdot 2a)\right)$$

$$+ \left(0 + a^2 \cdot (t \cdot a)\right)$$

$$I_y = \frac{8a^3 \cdot t}{3}$$

$$I_z = \sum \left(I_{z,i} + y_i^2 \cdot A_i\right)$$

$$I_z = \left(\frac{a^3 \cdot t}{12} + \left(\frac{a}{2} + \frac{t}{2}\right)^2 \cdot t \cdot a\right) + \left(0 + 0 \cdot 2at\right)$$

$$+ \left(\frac{a^3 \cdot t}{12} + \left(\frac{a}{2} + \frac{t}{2}\right)^2 \cdot t \cdot a\right)$$

$$\boxed{I_z = \frac{2a^3 \cdot t}{3}}$$

$$I_{yz} = \sum \left(I_{yz_i} - y_i \cdot z_i \cdot A_i \right)$$

$$I_{yz} = \left(0 - a \cdot \left(\frac{a}{2} + \frac{t}{2}\right) \cdot t \cdot a\right) + \left(0 - 0 \cdot (a \cdot t)\right) + \left(0 - (-a) \cdot \left(-\frac{a}{2} - \frac{t}{2}\right) \cdot t \cdot a\right)$$

$$t << a$$

$$I_{yz} = -\frac{a^3 \cdot t}{2} - \cancel{\frac{a \cdot t^2}{2}} - \frac{a^3 \cdot t}{2} - \cancel{\frac{t^2 \cdot a}{2}}$$

$$\boxed{I_{yz} = -a^3 t}$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

$$I_{1,2} = \frac{5}{3} a^3 \cdot t \pm \sqrt{2 \underline{\underline{(a^3 \cdot t)^2}}} \quad \rightarrow \sqrt{2} \cdot a^3 \cdot t$$

$$\boxed{\begin{aligned} I_1 &= 3{,}08\, a^3 t \\ I_2 &= 0{,}2\overline{5}\, a^3 \cdot t \end{aligned}} \longrightarrow$$

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$$\tan(2\alpha) = \frac{2 I_{yz}}{I_y - I_z}$$

$$2\alpha = \tan^{-1}(1) = -45^\circ$$

$$= \boxed{\alpha = -22{,}5^\circ}$$

neg. Drehrichtung

$$I_\eta = \frac{1}{2}(I_y + I_z) + \frac{1}{2}(I_y - I_z) \cdot \cos(2\alpha) + I_{yz} \cdot \sin(2\alpha)$$

$$I_\eta = 3{,}08\, a^3 \cdot t = I_1$$

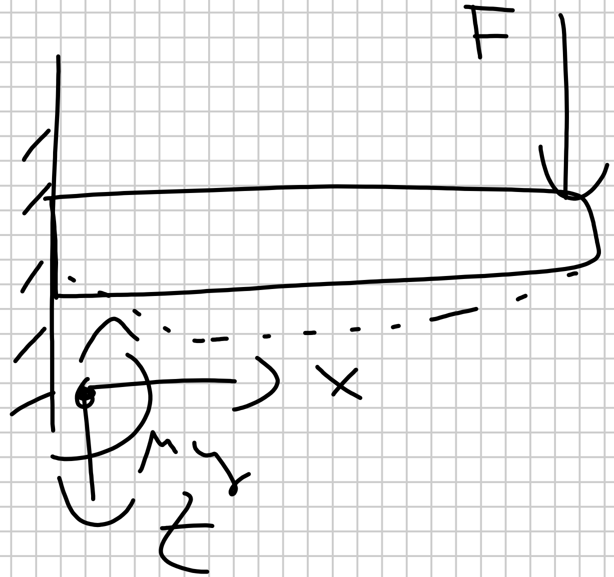
$I_1 \rightarrow -22{,}5°$

$I_2 \rightarrow -22{,}5° + 90° = 67{,}5°$

$$E v(x)'' = \frac{M_z \cdot I_y - M_y \cdot I_{yz}}{I_y \cdot I_z - I_{yz}^2}$$

$$E w(x)'' = \frac{M_z \cdot I_{yz} - M_y \cdot I_z}{I_y \cdot I_z - I_{yz}^2}$$

$$\boxed{M_z = 0}$$



$$\rightarrow: A_h = 0$$

$$\uparrow: A_v - F = 0$$

$$\boxed{F = A_v}$$

$$\circlearrowleft A: -M_A - F \cdot l = 0$$

$$M_A = -F \cdot l$$

$$\circlearrowleft S: \quad -M_A - A_v \cdot x + M_y = 0$$

$$M_y = M_A + A_v \cdot x$$

$$\boxed{M_y = -F \cdot l + F \cdot x} \rightarrow F \cdot (x - l)$$

y-Richtung

$$EIv'' = -\frac{F(x-l)\cdot(-a^3\cdot t)}{\frac{8}{3}\cdot a^3\cdot t\cdot\frac{2}{3}\cdot a^3\cdot t-(a^3\cdot t)^2}$$

$$Ev'' = \frac{9}{7}\cdot\frac{F(x-l)}{a^3\cdot t}$$

$$Ev' = \frac{9F}{7a^3\cdot t}\cdot\int(Fx - F\cdot l)\,dx$$

$$Ev' = \frac{9}{7a^3 \cdot t} \cdot \int (Fx - F\ell) \cdot dx$$

$$Ev' = \frac{9}{7a^3 \cdot t} \cdot \left(\frac{1}{2}Fx^2 - F\ell \cdot x\right) + C_1$$

$$Ev = \frac{9}{7a^3 \cdot t} \cdot \int \left(\frac{1}{2}Fx^2 - F\ell \cdot x\right) dx + \int C_1 \cdot dx$$

$$Ev = \frac{9}{7a^3 \cdot t} \cdot \left(\frac{1}{6}F \cdot x^3 - \frac{1}{2}F \cdot \ell x^2\right) + C_1 \cdot x + C_2$$

$$\boxed{\begin{aligned} v &= 0\,,\ v' = 0 \\ w &= 0\,,\ w' = 0 \end{aligned}}$$

<u>$v' = 0$ für $x = 0$</u>:

$$E \cdot 0 - \frac{9}{7 \cdot a^3 \cdot t} \cdot \left(\frac{1}{2} F \cdot 0^2 - F \cdot l \cdot 0\right) + C_1 \quad \Rightarrow \boxed{C_1 = 0}$$

$$E \cdot 0 = \frac{9}{7 a^3 \cdot t} \cdot \left(\frac{1}{6} \cdot F \cdot 0^3 - \frac{1}{2} F \cdot l \cdot 0^2\right) + 0 \cdot 0 + C_2$$

$$\Rightarrow \boxed{C_2 = 0}$$

$$E_V = \frac{9}{7 a^3 t} \cdot \left(\frac{1}{6} \cdot F x^3 - \frac{1}{2} \cdot F \cdot l \cdot x^2 \right)$$

$$\boxed{E_V = \frac{9F}{7 a^3 t} \cdot \left(\frac{1}{6} \cdot x^3 - \frac{1}{2} \cdot l \cdot x^2 \right)}$$

$$x \leq l$$

$$\frac{1}{2} \cdot l \cdot x^2 > \frac{1}{6} \cdot x^3 \Rightarrow$$ Durchbiegung in negative y-Richtung

$$E w'' = -\frac{F(x-l)\cdot\frac{4}{3}(a^3\cdot t)}{\frac{8}{3}a^3\cdot t\cdot\frac{2}{3}a^3 t-(a^3\cdot t)^2}$$

$$E w'' = \frac{F(x-l)\cdot\frac{2}{3}(a^3\cdot t)}{\frac{8}{3}a^3\cdot t\cdot\frac{2}{3}\cdot a^3\cdot t\cdot(a^3\cdot t)^2}$$

$$E w'' = -\frac{6F(x-l)}{7a^3\cdot t}$$

$$E w'' = -\frac{6}{7a^3 t} \cdot \int (F x - F l) \cdot dx$$

$$E w' = -\frac{6}{7a^3 t} \cdot \left(\frac{1}{2} F x^2 - F \cdot l \cdot x\right) + C_3$$

$$E w = -\frac{6}{7a^3 t} \cdot \left(\frac{1}{6} \cdot F \cdot x^3 - \frac{1}{2} \cdot l \cdot x^2\right) + C_3 \cdot x + C_4$$

$$\boxed{w = 0 \; , \; w' = 0 \qquad x = 0}$$

$$\rightarrow \boxed{\begin{array}{l} C_4 = 0 \\ C_3 = 0 \end{array}}$$

$$E w = -\frac{6F}{7a^3 \cdot t} \cdot \left(\frac{1}{6} x^3 - \frac{1}{2} \cdot l \cdot x^2 \right)$$

$$\frac{1}{6} x^3 < \frac{1}{2} \cdot l \cdot x$$

$\Rightarrow$ Durchbiegung in positive z-Richtung