Webinar: Höhere Mathematik 1
Thema: Vektorrechnung

Notiztitel 12.07.2016

1) $\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$

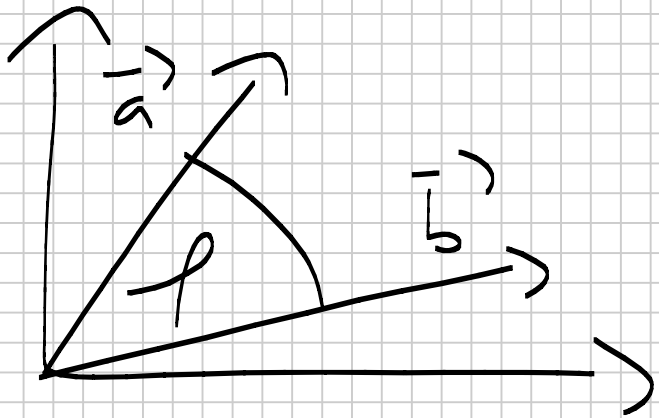
$= 2 \cdot 1 + 1 \cdot 2 + 0 \cdot 1 = 4$

$\vec{a} \cdot \vec{c} = a_1 \cdot c_1 + a_2 \cdot c_2 + a_3 \cdot c_3$

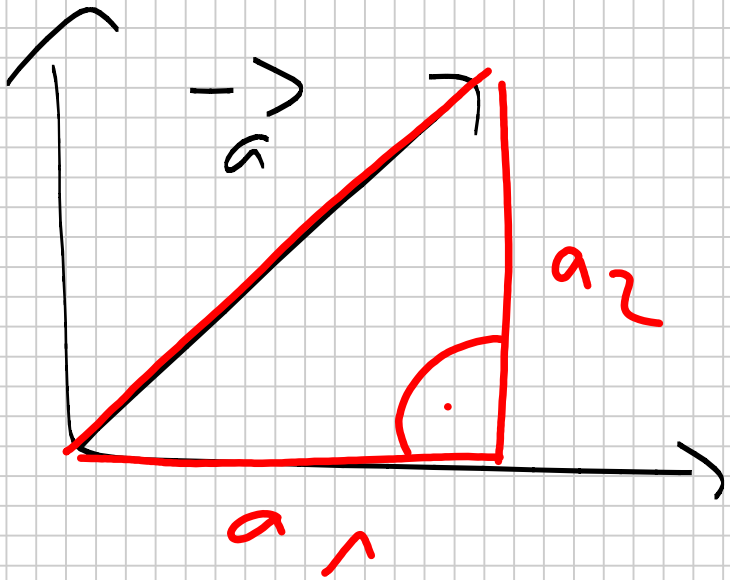
$2 \cdot 3 - 1 \cdot 2 + 0 \cdot 0 = 8$

1b) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\varphi)$

$$\varphi = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)$$



Länge:

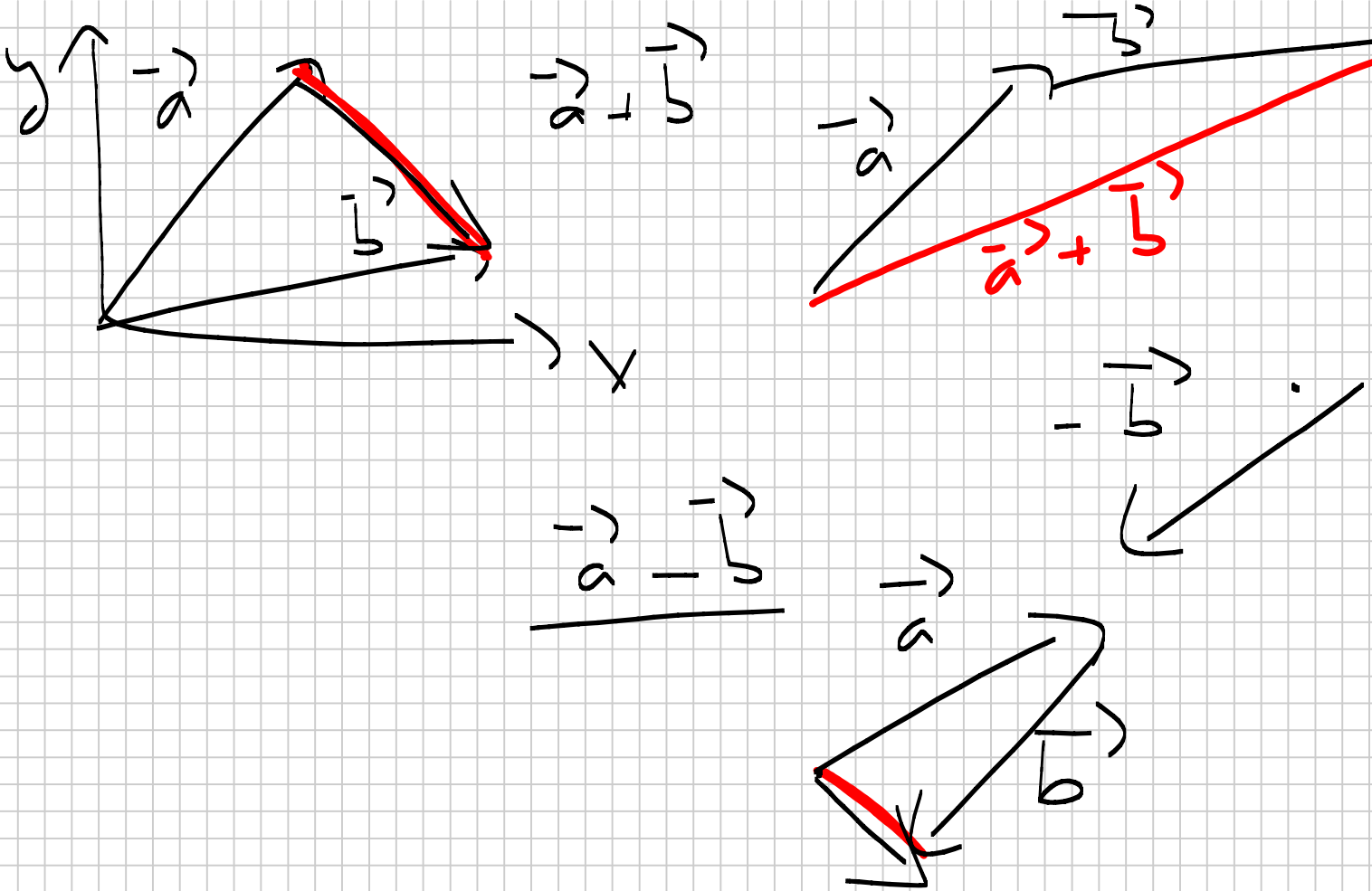


$$\sqrt{a_1^2 + a_2^2} = |\vec{a}|$$

$$|\vec{a}| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{5}$$

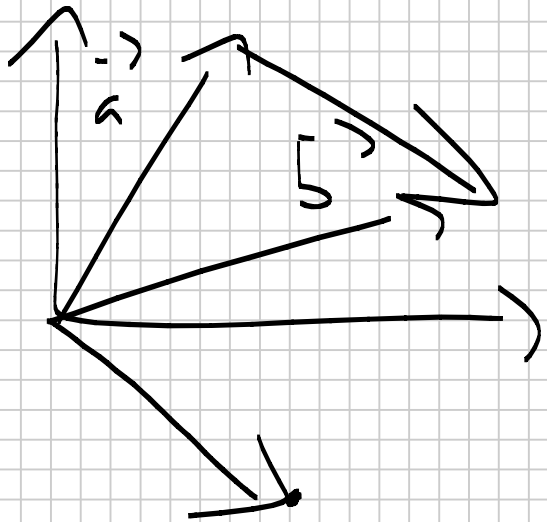
$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$\varphi = \cos^{-1}\left(\frac{4}{\sqrt{5} \cdot \sqrt{6}}\right) = 43{,}09^\circ$$



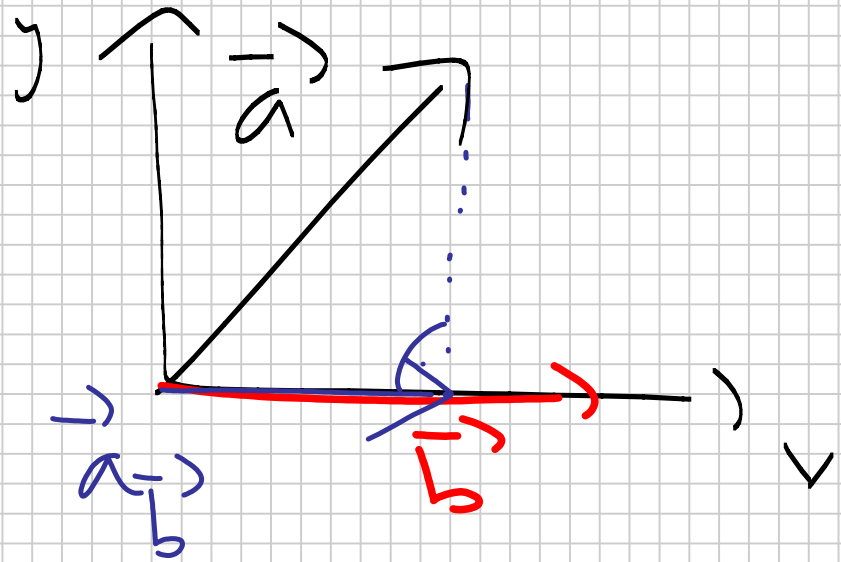
$$|\vec{a} - \vec{b}| \qquad |\vec{b} - \vec{a}|$$

$$\vec{a} - \vec{b} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$$



$$|\vec{a} - \vec{b}| = \sqrt{1^2 + 2^2 + 6^2} = \sqrt{41} = 6{,}4$$

3)



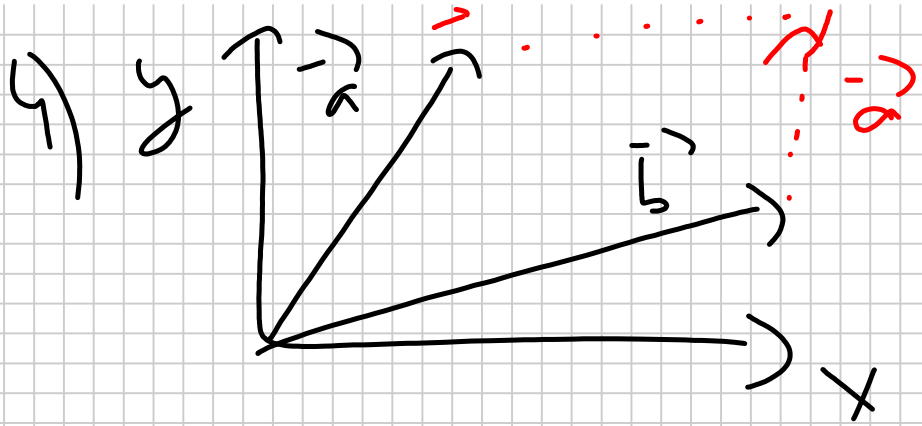
$$\vec{a}_{\vec{b}} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \cdot \vec{b}$$

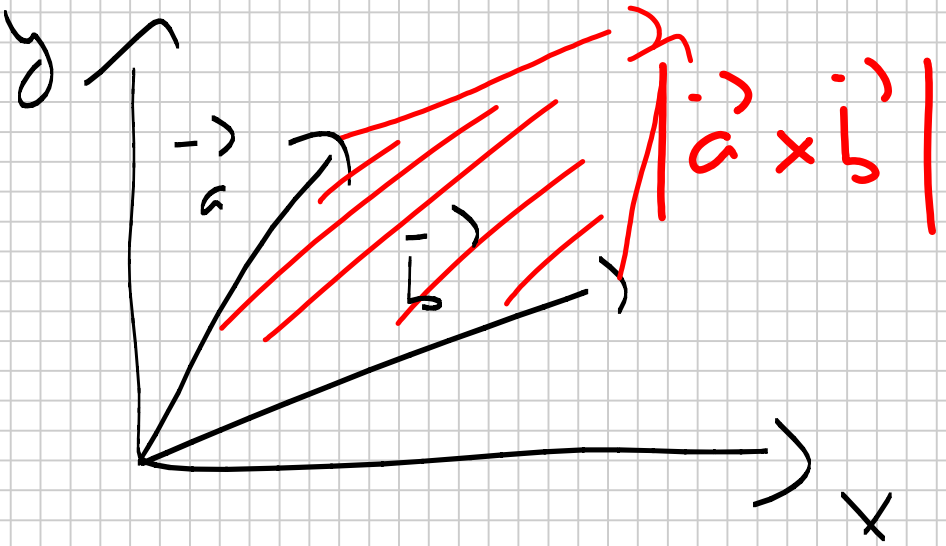
$$\vec{a} \cdot \vec{b} = 5$$

$$|\vec{b}| = \sqrt{2}$$

$$\vec{a}_{\vec{b}} = \frac{5}{\sqrt{2}^2} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2{,}5 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2{,}5 \\ 2{,}5 \\ 0 \end{pmatrix}$$

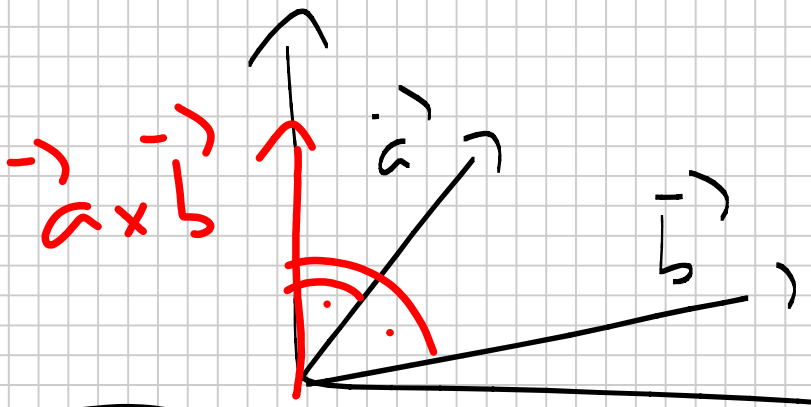
$$\vec{a}_{\vec{b}} = 3{,}54 = \sqrt{2{,}5^2 + 2{,}5^2 + 0^2}$$





$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2 \cdot b_3 - a_3 \cdot b_2 \\ a_3 \cdot b_1 - a_1 \cdot b_3 \\ a_1 \cdot b_2 - a_2 \cdot b_1 \end{pmatrix}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} 3 \cdot 5 - 2 \cdot 1 \\ 2 \cdot 3 - 1 \cdot 5 \\ 1 \cdot 1 - 3 \cdot 3 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \\ -8 \end{pmatrix}$$



Fläche:

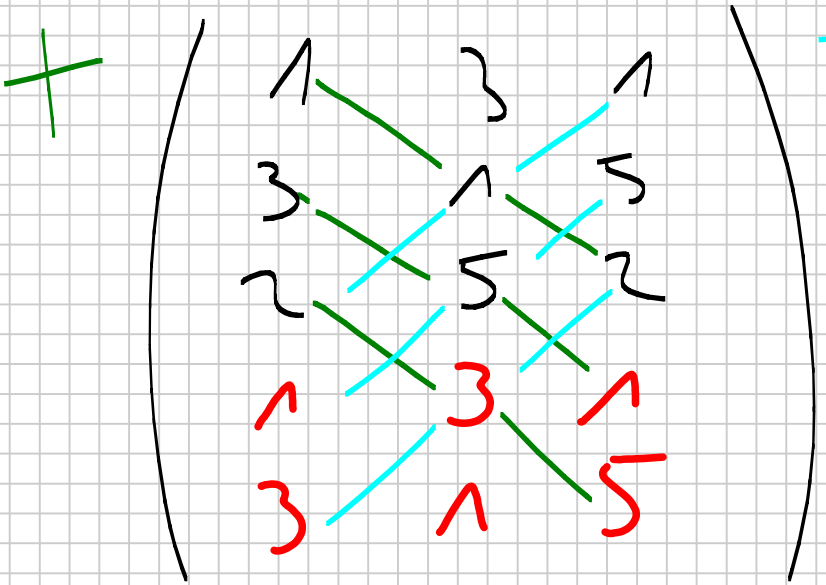
$$|\vec{a} \times \vec{b}| = \sqrt{13^2 + 1^2 + (-8)^2} = \sqrt{234}$$

$$= 15{,}30$$

5) $(\vec{a}, \vec{b}, \vec{c}) = \vec{a} \cdot (\vec{b} \times \vec{c})$

$$\vec{b} \times \vec{c} = \begin{pmatrix} b_2 \cdot c_3 - b_3 \cdot c_2 \\ b_3 \cdot c_1 - b_1 \cdot c_3 \\ b_1 \cdot c_2 - b_2 \cdot c_1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 2 - 5 \cdot 5 \\ 5 \cdot 1 - 3 \cdot 2 \\ 3 \cdot 5 - 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} -23 \\ -1 \\ 14 \end{pmatrix}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -23 \\ -1 \\ 14 \end{pmatrix} = 2$$



$$+1\cdot\uparrow\cdot 2+3\cdot 5\cdot\uparrow+2\cdot 3\cdot 5-\uparrow\cdot\uparrow\cdot 2$$
$$-5\cdot 5\cdot\uparrow-2\cdot 3\cdot 3=2$$