1a) $A_1 = 16a^2$, $z_{S_1} = a$

$A_2 = 4a^2$, $z_{S_2} = 4a$

$y_S = 0$ wg. Symmetrie

$$\Rightarrow z_S = \frac{\sum_{i=1}^{2} (A_i \cdot z_{S_i})}{\sum_{i=1}^{2} A_i} = \frac{A_1 \cdot z_{S_1} + A_2 \cdot z_{S_2}}{A_1 + A_2}$$

$$\Rightarrow \frac{16a^3 + 16a^3}{20a^2} = \frac{32}{20} a = \underline{\underline{\frac{8}{5} a}}$$

1b) $I_{yz} = 0$ (Symmetrie)

$$I_{y_1} = \frac{1}{12} \cdot 8a \cdot (2a)^3 = \frac{64}{12} a^4 = \frac{16}{3} a^4$$

$$I_{y_2} = \frac{1}{12} a \cdot (4a)^3 = \frac{16}{3} a^4$$

$$I_{z_1} = \frac{1}{12} \cdot 2a \cdot (8a)^3 = \frac{256}{3} a^4$$

$$I_{z_2} = \frac{1}{12} \cdot 4a \cdot a^3 = \frac{1}{3} a^4$$

$$I_y = \sum_{i=1}^{2} I_{y_i} + \sum_{i=1}^{2} (z_{S_i} - z_S)^2 \cdot A_i$$

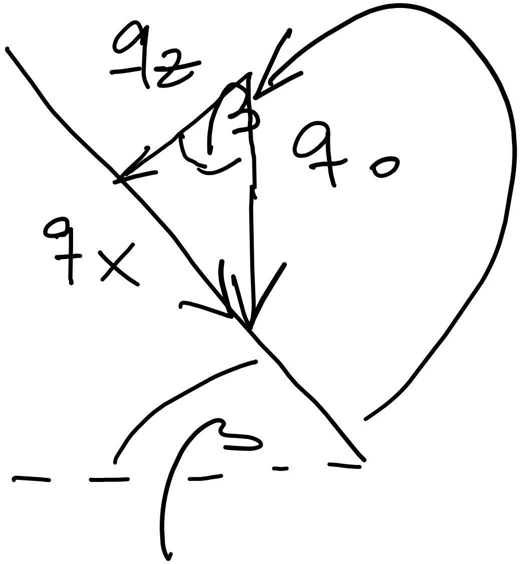
$$= \frac{32}{3} a^4 + \left(+\frac{3}{5} a - \frac{8}{5} a\right)^2 \cdot 16a^2 + \left(\frac{20}{5} a - \frac{8}{5} a\right)^2 \cdot 4a^2$$

$$\Rightarrow \frac{32}{3}a^4 + \frac{9}{25}a^2 \cdot 16a^2 + \frac{144}{25}a^2 \cdot 4a^2$$

$$= \frac{32}{3}a^4 + \frac{144}{5}a^4 = \underline{\underline{\frac{592}{15}a^4}}$$

$$I_z = I_{z_1} + I_{z_2} + 0 = \left(\frac{256}{3} + \frac{1}{3}\right)a^4 = \underline{\underline{\frac{257}{3}a^4}}$$

c)

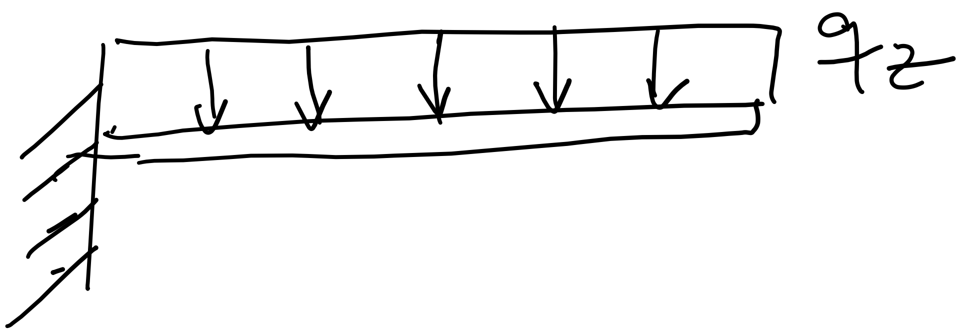


$$\Rightarrow \quad q_z = q_0 \cdot \cos\beta$$
$$q_x = q_0 \cdot \sin\beta$$

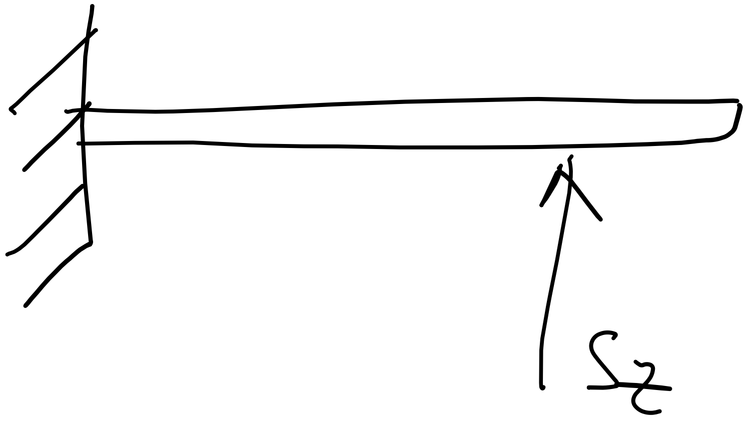
zu d)

$$\Rightarrow \quad S_z = S \cdot \cos\beta$$
$$S_x = S \cdot \sin\beta$$

d) Kräfteskizze:



und



mit $q_z = q_0 \cdot \cos\beta$

mit $S_z = S \cdot \cos\beta$

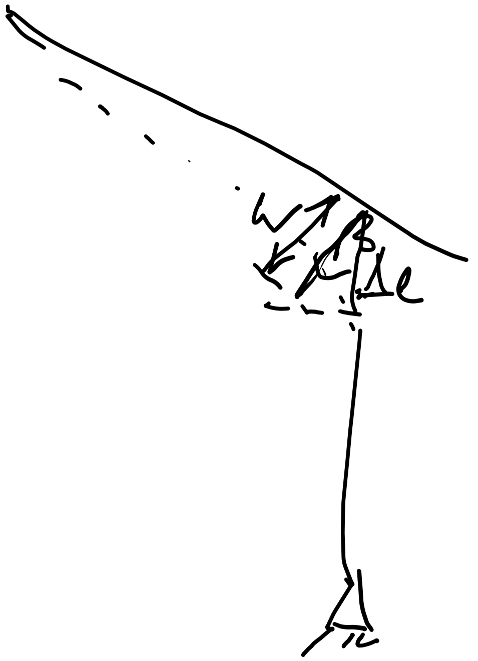
$\underline{\overline{\mathrm{I}}}$

$\underline{\underline{\overline{\mathrm{II}}}}$

$$w_{ges}(2l) = w_I(2l) + w_{II}(2l) = w$$

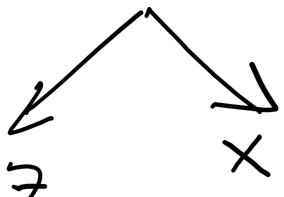
$$\Rightarrow \Delta l = -\frac{S \cdot l}{E_S \cdot A_S} + \alpha_S \cdot \Delta T \cdot l$$

$$\Rightarrow |\Delta l| = \underbrace{w_{\cancel{I}} \cdot \cos\beta}_{= w_I} \Rightarrow w_{\cancel{I}} = \frac{|\Delta l|}{\cos\beta}$$

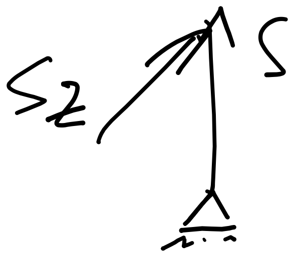


e) $$w_I(2l) = \frac{q_z}{24\cdot EI}\cdot\left(6\cdot\left(\frac{5}{2}l\right)^2\cdot(2l)^2 - 4\cdot\left(\frac{5}{2}l\right)\cdot\underbrace{(8l^3)}_{(2l)^3} + (2l)^4\right)$$

$$w_{II}(2l) = -\frac{S_z(2l)^3}{3EI} = -\frac{S\cdot\cos\beta\cdot 8l^3}{3EI}$$



$$\Rightarrow w_I(2l) = \frac{q_z}{24EI}\cdot\left(\frac{\overset{150}{\cancel{6\cdot 25}}}{\cancel{4}}l^2\cdot\cancel{4}l^2 - 10l\cdot 8l^3 + 16l^4\right)$$



$$= \frac{q_z}{24EI}\cdot\left(150l^4 - 80l^4 + 16l^4\right)$$

$$= \frac{q_0\cdot\cos\beta}{24EI}\cdot 86l^4$$

$$\Rightarrow w = \underline{\underline{\frac{q_0\cdot\cos\beta}{24EI}\cdot 86l^4 - \frac{S\cdot\cos\beta\cdot 64l^3}{24E\cdot I}}}$$

f) $w = \dots$

$$w = \frac{|\Delta l|}{\cos\beta}$$

$$\Rightarrow \frac{q_0 \cdot \cancel{\cos}^{2}\beta}{24EI} \cdot 86 l^4 - \frac{S \cdot \cos^{2}\beta \cdot \cancel{16}^{64 l^3}}{24EI} = \frac{\frac{S \cdot l}{E_S \cdot A_S} - \alpha_S \Delta T \cdot l}{\cancel{\cos\beta}}$$

$$\Leftrightarrow \frac{q_0 \cdot \cos^2\beta \cdot 86 \cancel{l^4}^{3}}{24EI} + \alpha_S \cdot \Delta T \cdot \cancel{l} = \frac{S \cdot \cancel{l}}{E_S A_S} + \frac{S \cos^2\beta \cdot 64 \cancel{l^3}^{2}}{24EI}$$

$$\Leftrightarrow \frac{q_0 \cos^2\beta \, 86 l^3}{24EI} + \alpha_S \cdot \Delta T = S\left(\frac{1}{E_S A_S} + \frac{\cos^2\beta \cdot 64 l^2}{24EI}\right)$$

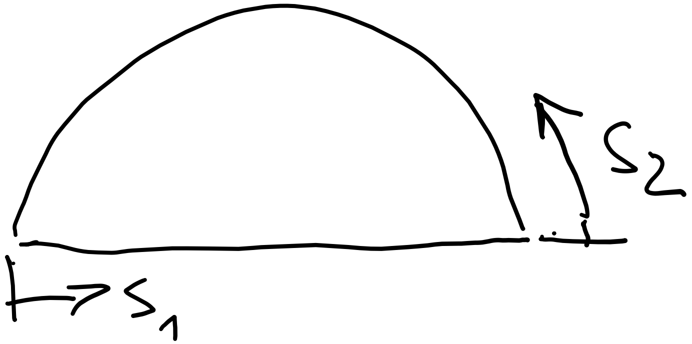
$$\Rightarrow S = \frac{\dfrac{q_0 \cos^2\beta \cdot 86 l^3}{24EI} + \alpha_S \cdot \Delta T}{\dfrac{1}{E_S A_S} + \dfrac{\cos^2\beta \cdot 64 l^2}{24EI}}$$

2a)

$$\varphi = \frac{M_T \cdot l}{G \cdot I_T} \quad , \quad M_T = F \cdot r$$

$$G = \frac{E}{2 \cdot (1+\nu)}$$

$$I_T = \frac{4 \cdot A_m^2}{\oint_C \frac{ds}{t(s)}} \quad , \quad A_m = \frac{\pi r^2}{2} \Rightarrow A_m^2 = \frac{\pi^2 \cdot r^4}{4}$$



$$\Rightarrow \oint \frac{ds}{t(s)} = \int_0^{2r} \frac{ds_1}{\delta} + \int_0^{\pi \cdot r} \frac{ds_2}{\delta} = \frac{1}{d} \cdot (2r + \pi r)$$

$$= \frac{r}{d} (2 + \pi)$$

$$\Rightarrow I_T = \frac{\frac{\cancel{4} \cdot \pi^2 r^4}{\cancel{4}}}{\frac{r}{\delta} \cdot (2+\pi)} = \frac{\pi^2 r^{\cancel{4}\,3}}{2+\pi} \cdot \frac{\delta}{\cancel{r}} = \frac{\pi^2 \cdot r^3 \cdot \delta}{2+\pi}$$

$$\Rightarrow \varphi_{zul} \geq \frac{F \cdot r \cdot l}{\frac{E}{2 \cdot (1+\nu)} \cdot \frac{\pi^2 \cdot r^3 \cdot \delta}{2+\pi}} = \frac{F \cdot \cancel{r} \cdot l \cdot 2 \cdot (1+\nu) \cdot (2+\pi)}{E \cdot \pi \cdot r^{\cancel{3}\,2} \cdot \delta}$$

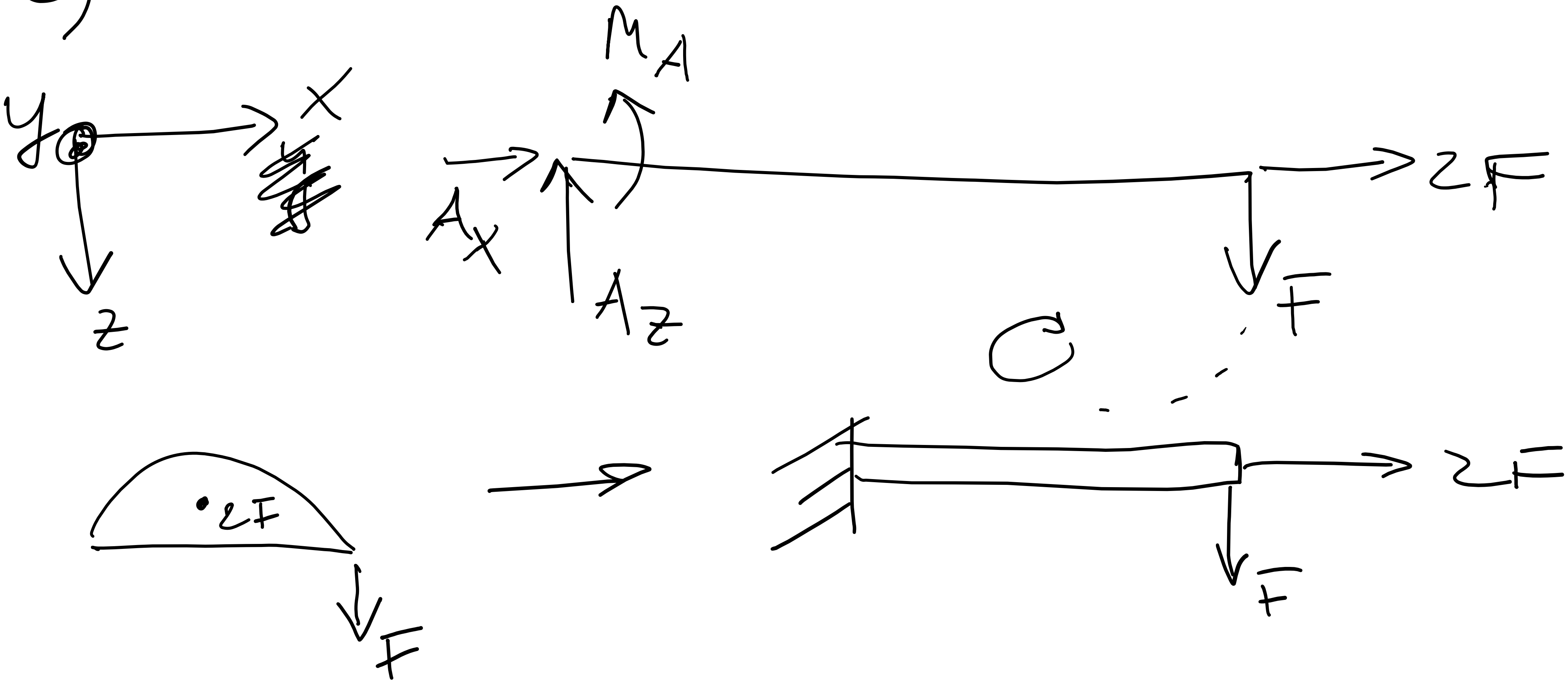
$$\Leftrightarrow \delta \geq \frac{F \cdot l \cdot 2(1+\nu) \cdot (2+\pi)}{E \cdot \pi \cdot r^2 \cdot \varphi_{zul}}$$

b) $\tau_{max} = \frac{M_T}{2 \cdot A_m \cdot t_{min}} = \frac{F \cdot \cancel{r}}{\cancel{2} \cdot \frac{\pi r^{\cancel{2}}}{\cancel{2}} \cdot \delta} = \frac{F}{\pi \cdot r \cdot \delta}$

mit t_{min} = geringste Wandstärke = δ

mit $\delta = \ldots$

c)

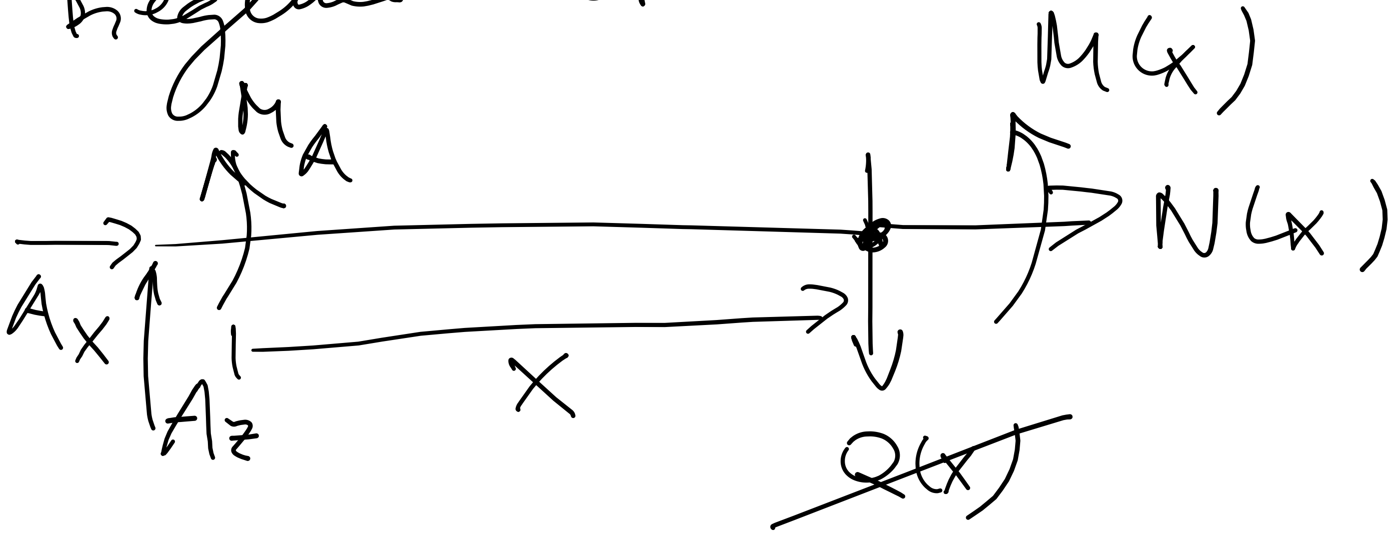


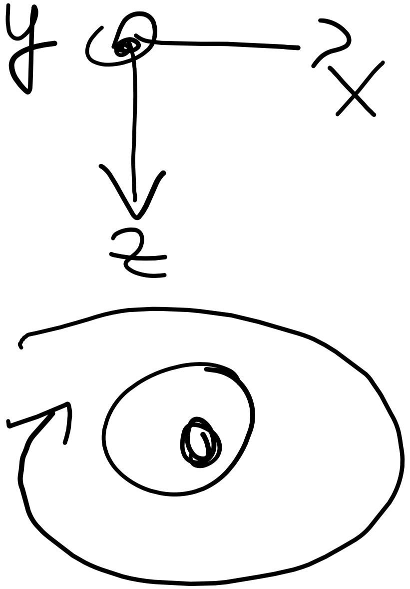
$$\sum F_x = 0 \Rightarrow A_x = -2F$$

$$\sum F_z = 0 \Rightarrow A_z = F$$

$$\sum M_A^{+} = 0 \Rightarrow M_A - F \cdot l = 0 \Leftrightarrow M_A = F \cdot l$$

Biegemoment:





$$\Rightarrow N(x) = -A_x = 2F$$

$$\sum M(x) = M(x) + M_A - A_z \cdot x = 0$$

$$\Leftrightarrow M(x) = A_z \cdot x - M_A = F \cdot x - F \cdot l$$

$$= F(x-l) \Rightarrow M_{max} = F \cdot l$$

$$\sigma(x,z) = \underbrace{\frac{M_y(x)}{I_y}}_{\sigma_b} \cdot z + \underbrace{\frac{N(x)}{A}}_{\sigma_{z/D}}$$

$$\rightarrow \sigma_{max} = \sigma(x=0, z=z_0) = -\frac{F \cdot l}{I_y} \cdot z_0 + \frac{2 \cdot F \cdot l}{A}$$

mit $A = A_{profil}$ $\rightarrow$ und $z_0 = -\delta - r + z_s$

$$\sigma_v = \sqrt{\sigma_{\cancel{z}\,max}^2 + 3 \cdot \tau_{max}^2} \quad \text{mit } \sigma = \sigma_b + \sigma_{z/D}$$

$$\text{und } \tau = \tau_t + \underbrace{\cancel{\tau_s}}_{=0}$$

$$= \sqrt{\left(-\frac{F \cdot l}{I_y} \cdot z_0 + \frac{2 \cdot F \cdot l}{A}\right)^2 + 3 \cdot \left(\frac{F}{\pi \cdot r \cdot \delta}\right)^2}$$

$$\text{mit } \delta = \ldots$$

d) $\varphi_{Riß} = \dfrac{M_T \cdot l}{G \cdot I_T}$

mit $\delta_{min} = \delta = \dots$

Für dünnwandige Querschnitte gilt:

$$I_T = \sum_{i=1}^{n=2} \frac{1}{3} l_i \cdot t_i^3 = \frac{\delta_{min}^3}{3} \cdot r(2+\pi)$$

$$\varphi_{Riß} = \frac{F \cdot l \cdot \cancel{r} \cdot 3}{G \cdot \delta_{min}^3 \cdot \cancel{r}(2+\pi)} = \underline{\underline{\frac{3 \cdot F \cdot l}{G \cdot \delta_{min}^3 \cdot (2+\pi)}}}$$

mit $G = \dots$

e) $\tau_{max} = \tau_{Riß} = \dfrac{M_T}{I_T \cdot \cancel{t}} \cdot t_{max}$

$$\Rightarrow \frac{F \cdot \cancel{l}}{\frac{\delta_{min}^{\cancel{3}\,2}}{3} \cdot \cancel{l}\,(2+\tilde{u})} \cdot \cancel{\delta_{min}} = \underline{\underline{\frac{3 \cdot F}{\delta_{min}^{2}\,(2+\tilde{u})}}}$$

mit $t_{max} = t_{min} = \delta_{min}$